Eur. Phys. J. A **17**, 493–498 (2003)
DOI 10.1140/epja/i2002-10197-2

THE EUROPEAN PHYSICAL JOURNAL A

Generalized parton distributions

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Received: 1 November 2002 /
Published online: 15 July 2003 –

Abstract. An introductory review of generalized parton distributions (GPDs) is given.

PACS. 13.40.Gp Electromagnetic form factors – 13.60.Fz Elastic and Compton scattering – 13.60.Hb Total and inclusive cross-sections (including deep-inelastic processes)

1 Introduction

The present-day situation in hadron physics can be briefly characterized in the following way.
i) We know what are the fundamental particles from which the hadrons are built: quarks and gluons.
ii) Quark-gluon interactions are described by quantum chromodynamics (QCD), and the QCD Lagrangian is known.
iii) But we still need to understand how QCD works, — *i.e.*, to understand hadronic structure in terms of quark and gluon fields.

Projecting quark and gluon fields onto hadronic states $|P\rangle$ gives matrix elements like $\langle 0 \,|\, \bar{q}_\alpha(z_1)\, q_\beta(z_2) \,|\, P \rangle$ (for mesons) which can be interpreted as hadronic wave functions. In the light-cone formalism [1], a hadron is described by its Fock components in the infinite momentum frame. For the nucleon, the Fock decomposition can be schematically written as $|P\rangle = |qqq\rangle + |qqqG\rangle + |qqq\bar{q}q\rangle + \ldots$. In principle, solving the bound-state equation $H|P\rangle = E|P\rangle$ one should get the wave function $|P\rangle$ containing complete information about the hadron structure. In practice, the equation (involving an infinite number of Fock components) has not been solved. Moreover, the LC wave functions are not directly accessible experimentally. The way out in this situation is the description of hadron structure in terms of phenomenological functions. Among "old" functions used for a long time we can list form factors, usual parton densities, and distribution amplitudes. The new functions, generalized parton distributions [2–4] (for a recent review see [5]), are hybrids of form factors, parton densities and distribution amplitudes. Furthermore, "old" functions are limiting cases of "new" ones.

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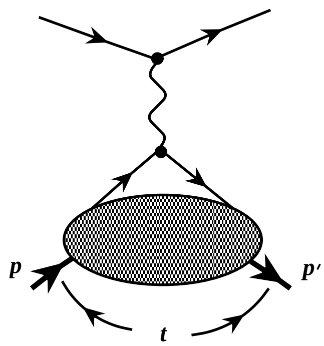

Fig. 1. Elastic eN scattering in one-photon approximation.

2 Form factors

Form factors are defined through matrix elements of electromagnetic and weak currents between hadronic states. In particular, the nucleon electromagnetic form factors measurable through elastic eN scattering (fig. 1) are given by

$$\langle p' \,|\, J^\mu(0) \,|\, p \rangle = \bar{u}(p') \left[\gamma^\mu F_1(t) + \frac{r^\nu \sigma^{\mu\nu}}{2m_N} F_2(t) \right] u(p) \ , \tag{1}$$

where $r = p - p'$ is the momentum transfer and $t = r^2$. The electromagnetic current is given by the sum of its flavor components $J^\mu(z) = \sum_a e_a \bar{\psi}_a(z)\gamma^\mu \psi_a(z)$. The form factors can also be written as sums over a, *e.g.*, $F_1(t) = \sum_a e_a F_{1a}(t)$ for the helicity nonflip form factor $F_1(t)$. At $t = 0$, these functions have well-known limiting values. In particular, $F_1(t=0) = e_N = \sum_a N_a e_a$ gives the total electric charge of the nucleon (N_f is the number of valence quarks of flavor a) and $F_2(t=0) = \kappa_N$ gives its anomalous magnetic moment.

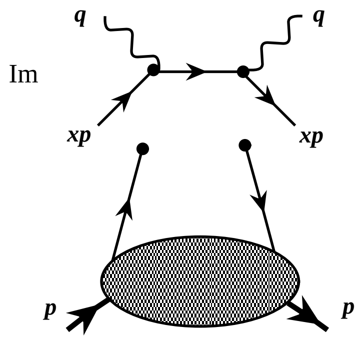

Fig. 2. Lowest-order pQCD factorization for DIS.

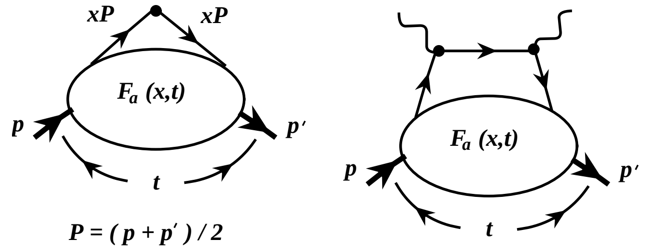

Fig. 3. Form factor and wide-angle Compton scattering amplitude in terms of nonforward parton densities.

3 Usual parton densities

The parton densities are defined through forward matrix elements of quark/gluon fields separated by lightlike distances. In the unpolarized case we have

$$\langle p \,|\, \bar{\psi}_a(-z/2)\gamma^\mu \psi_a(z/2) \,|\, p \rangle\big|_{z^2=0} = 2p^\mu \int_0^1 \left[e^{-ix(pz)} f_a(x) - e^{ix(pz)} f_{\bar{a}}(x) \right] \mathrm{d}x \ . \quad (2)$$

In the local limit $z = 0$, operators in this definition convert into vector currents entering into the definition of form factors. Since $t = 0$ for the forward matrix element, we obtain the sum rule for the numbers of valence quarks

$$\int_0^1 [f_a(x) - f_{\bar{a}}(x)]\,\mathrm{d}x = N_a \ . \quad (3)$$

The definition of parton densities has the form of a plane-wave decomposition. This observation allows to give the momentum space interpretation: $f_{a(\bar{a})}(x)$ is the probability to find a ($\bar{a}$)-quark with momentum xp inside a nucleon with momentum p. The classic process to access the usual parton densities is deep inelastic scattering (DIS) $\gamma^* N \to X$. Via the optical theorem its cross-section is given by the imaginary part of the forward virtual Compton scattering amplitude. When the spacelike momentum transfer q, $q^2 \equiv -Q^2$, is sufficiently large, perturbative QCD factorization works. At the leading order, one deals with the handbag diagram (fig. 2). Through simple algebra $\frac{1}{\pi}\mathrm{Im}\ 1/(q+xp)^2 \approx \delta(x - x_{\mathrm{Bj}})/2(pq)$ one finds that DIS measures parton densities at $x = x_{\mathrm{Bj}}$, when the parton momentum fraction equals the Bjorken variable $x_{\mathrm{Bj}} = Q^2/2(pq)$. Comparing parton densities to form factors, we note that the latter have a point vertex instead of a lightlike separation, and $p \neq p'$.

4 Nonforward parton densities

"Hybridization" of different parton distributions is the key idea of the GPD approach. Let us combine form factors with parton densities and write the flavor components $F_{1a}(t)$ of form factors as integrals over the momentum fraction x:

$$F_{1a}(t) = \int_0^1 [\mathcal{F}_a(x,t) - \mathcal{F}_{\bar{a}}(x,t)]\,\mathrm{d}x \ . \quad (4)$$

In the forward limit $t = 0$, the new objects, nonforward parton densities $\mathcal{F}_{a(\bar{a})}(x,t)$ (NPDs), coincide with the usual ("forward") densities: $\mathcal{F}_{a(\bar{a})}(x, t = 0) = f_{a(\bar{a})}(x)$. NPDs can be also treated as Fourier transforms of the impact parameter $b_\perp$ distributions $f(x, b_\perp)$ describing the variation of parton densities in the transverse plane.

An interesting question is the interplay between x and t dependence of $F_1(x,t)$. The simplest factorized ansatz $\mathcal{F}_a(x,t) = f_a(x)F_1(t)$ satisfies both the forward constraint: $\mathcal{F}_a(x, t = 0) = f_a(x)$ and the local constraint (4). The reality may be more complicated: light-cone wave functions with the Gaussian $k_\perp$ dependence $\Psi(x_i, k_{i\perp}) \sim \exp[-\sum_i k_{i\perp}^2/x_i\lambda^2]$ suggest $\mathcal{F}^a(x,t) = f_a(x)e^{\bar{x}t/2x\lambda^2}$. Taking $f_a(x)$ from existing parametrizations like GRV and adjusting λ^2 to provide a standard value of the quark intrinsic transverse momentum $\langle k_\perp^2 \rangle \approx (300\ \mathrm{MeV})^2$, gives a reasonable description of the proton form factor $F_1(t)$ in a wide range of momentum transfers $-t \sim 1 - 10\,\mathrm{GeV}^2$ [6].

The same nonforward parton densities appear in the handbag diagrams for the wide-angle real Compton scattering (fig. 3). The handbag term in this case is the product of a new form factor $R_V^a(t)$ given by the $1/x$ moment of $\mathcal{F}^a(x,t)$ and the amplitude of the Compton scattering off an elementary fermion. For the cross-section, this gives

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \left[\sum_a e_a^2 R_V^a(t)\right]^2 \frac{\mathrm{d}\sigma}{\mathrm{d}t}\bigg|_{\mathrm{KN}} \ , \quad (5)$$

where $\mathrm{d}\sigma/\mathrm{d}t|_{\mathrm{KN}}$ is the (Klein-Nishina) cross-section for the Compton scattering off an electron.

The predictions based on handbag mechanism dominance and NPDs [6,7] are in much better agreement with existing Cornell data than the predictions based on two-gluon hard exchange mechanism of asymptotic perturbative QCD: the predicted cross-section is too small in the latter case. The absolute normalization for predictions is settled by the form of the nonperturbative functions (NPDs in the handbag approach and nucleon distribution amplitudes in the pQCD approach) which were fixed by fitting the F_1 form factor data. Still, when there is an uncertain overall factor, it is risky to make

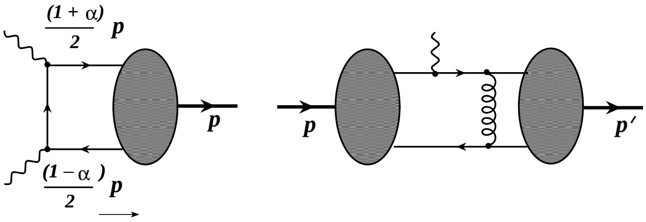

Fig. 4. Lowest-order pQCD factorization for $\gamma^*\gamma \to \pi^0$ transition amplitude and for the pion EM form factor.

strong statements. Remarkably, the perturbative QCD hard-scattering mechanism and soft handbag mechanism give drastically different predictions for the polarization asymmetry A_{LL} [7]. Experiment E-99-114 recently performed at Jefferson Lab [8] strongly favors the handbag mechanism that predicts the value close to the asymmetry for the scattering on a single quark.

5 Distribution amplitudes

Another example of nonperturbative functions describing the hadron structure are the distribution amplitudes (DAs). They can be interpreted as light-cone wave functions integrated over transverse momentum, or as $\langle 0 | \ldots | p \rangle$ matrix elements of light-cone operators. In the case of the pion we have

$$\langle\, 0\, |\, \bar\psi_d(-z/2)\gamma_5\gamma^\mu\psi_u(z/2)\, |\, \pi^+(p)\,\rangle\big|_{z^2=0} = \\ ip^\mu f_\pi \int_{-1}^{1} e^{-i\alpha(pz)/2}\varphi_\pi(\alpha)\,\mathrm{d}\alpha\,, \qquad (6)$$

with $x_1 = (1+\alpha)/2,\ x_2 = (1-\alpha)/2$ being the fractions of the pion momentum carried by the quarks. The distribution amplitudes describe the hadrons in situations when the pQCD hard-scattering approach is applicable to exclusive processes. The classic example is the process of $\gamma^*\gamma \to \pi^0$ transition (fig. 4). Its amplitude is proportional to the $1/(1-\alpha^2)$ moment of $\varphi_\pi(\alpha)$. The predictions for the $\gamma^*\gamma \to \pi^0$ form factor based on two competing models for the pion DA, the asymptotic $\varphi_\pi^{\rm as}(\alpha) = \frac{3}{4}(1-\alpha^2)$ and Chernyak-Zhitnitsky DA $\varphi_\pi^{\rm CZ}(\alpha) = \frac{15}{4}\alpha^2(1-\alpha^2)$ differ by factor of 5/3, which allows for an experimental discrimination between them.

Comparison with CLEO and CELLO data for the combination $Q^2F_{\gamma^*\gamma\pi^0}(Q^2)$ favors $\varphi^{\rm as}(\alpha)$. It is also worth noting that perturbative QCD works here from rather small values of momentum transfer $Q^2 \sim 2\,\mathrm{GeV}^2$. Another classic application of pQCD to exclusive processes is the pion electromagnetic form factor. With the asymptotic pion DA $\varphi_\pi^{\rm as}(\alpha)$, the hard pQCD contribution to $F_\pi(Q^2)$ is $(2\alpha_s/\pi)(0.7\,\mathrm{GeV}^2)/Q^2$, which is less than 1/3 of the experimental value. So, in this case we deal with the dominance of the competing soft mechanism that is described by nonforward parton densities, exactly in the same way as the proton $F_1^p(t)$ form factor discussed in the previous section.

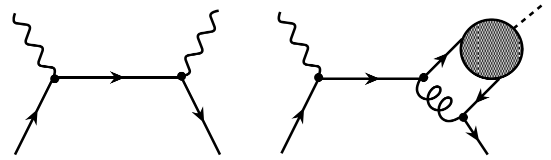

Fig. 5. Lowest-order hard subprocesses for deeply virtual photon and meson production.

6 Hard electroproduction processes

A more recent attempt to use perturbative QCD to extract new information about hadronic structure is the study of deep exclusive photon [3] or meson [4,9] electroproduction reactions. In the hard kinematics when both Q^2 and $s \equiv (p+q)^2$ are large while the momentum transfer $t \equiv (p-p')^2$ is small, one can use pQCD factorization which represents the amplitudes as a convolution of a perturbatively calculable short-distance amplitude and nonperturbative parton functions describing the hadron structure. The hard pQCD subprocesses in these two cases have different structure (fig. 5). Since the photon is a pointlike particle, the deeply virtual Compton scattering amplitude has a structure similar to that of the $\gamma^*\gamma\pi^0$ form factor: the pQCD hard term is of zero order in α_s, and there is no competing soft contribution. Thus, we can expect that pQCD works from $Q^2 \sim 2\,\mathrm{GeV}^2$. On the other hand, the deeply virtual meson production process is similar to the pion EM form factor: the hard term has $O(\alpha_s/\pi) \sim 0.1$ suppression factor. As a result, the dominance of the hard pQCD term may be postponed to $Q^2 \sim 5$–$10\ \mathrm{GeV}^2$.

One should also have in mind that the competing soft mechanism can mimic the same power law Q^2-behavior (just like in case of pion and nucleon EM form factors). Hence, a mere observation of a "right" power law behavior of the cross-section may be insufficient to claim that pQCD is already working. One should look at other characteristics of the reaction, especially its spin properties, to make strong statements about the reaction mechanism.

7 Deeply virtual Compton scattering and generalized parton distributions

It is convenient to visualize DVCS in the γ^*N center-of-mass frame, with the initial hadron and the virtual photon moving in opposite directions along the z-axis. Since the momentum transfer t is small, the hadron and the real photon in the final state also move close to the z-axis. This means that the virtual photon momentum $q = q' - x_{\rm Bj}p$ has the component $-x_{\rm Bj}p$ canceled by the momentum transfer r. In other words, the momentum transfer r has the longitudinal component $r^+ = x_{\rm Bj}p^+$, where $x_{\rm Bj} = Q^2/2(pq)$ is the DIS Bjorken variable. One can say that DVCS has a skewed kinematics in which the final hadron has the "plus"-momentum $(1-\zeta)p^+$ that is smaller than that of the initial hadron. In the particular case of DVCS, we have $\zeta = x_{\rm Bj}$.

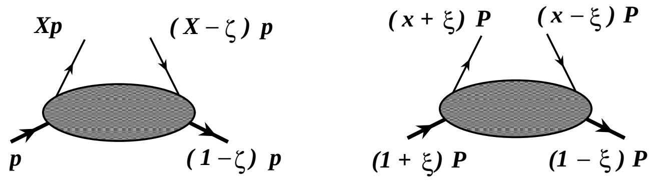

Fig. 6. Comparison of NFPDs and OFPDs.

The parton picture for DVCS has some similarity to that of DIS, with the main difference that the plus-momenta of the incoming and outgoing quarks in DVCS are not equal. They are Xp^+ and $(X-\zeta)p^+$. Another difference is that the invariant momentum transfer t in DVCS is nonzero: the matrix element of partonic fields is essentially nonforward.

Thus, the nonforward parton distributions (NFPDs) $\mathcal{F}_\zeta(X;t)$ describing the hadronic structure in DVCS depend on X, the fraction of p^+ carried by the outgoing quark, on ζ, the skewedness parameter characterizing the difference between initial and final hadron momenta, and on t, the invariant momentum transfer. In the forward $r=0$ limit, we have a reduction formula $\mathcal{F}^a_{\zeta=0}(X,t=0)=f_a(X)$ relating NFPDs with the usual parton densities. The nontriviality of this relation is that $\mathcal{F}_\zeta(X;t)$ appears in the amplitude of the exclusive DVCS process, while the usual parton densities are measured from the cross-section of the inclusive DIS reaction. Another limit for NFPDs is zero skewedness $\zeta=0$, where they correspond to nonforward parton densities $\mathcal{F}^a_{\zeta=0}(X,t)=\mathcal{F}^a(X,t)$. The local limit relates NFPDs to form factors

$$\int_0^1 \mathcal{F}^a_\zeta(X,t)\,\mathrm{d}X = F^a_1(t)\ . \tag{7}$$

The description in terms of NFPDs has the advantage of using the variables closest to those of the usual parton densities. However, the initial and final hadron momenta are not treated symmetrically in this scheme. Ji [3] proposed to use symmetric variables in which the plus-momenta of the hadrons are $(1+\xi)P^+$ and $(1-\xi)P^+$, and those of the active partons are $(x+\xi)P^+$ and $(x-\xi)P^+$, P being the average momentum $P=(p+p')/2$ (fig. 6).

To take into account the spin properties of the hadrons and of the quarks, one needs 4 off-forward parton distributions $H, E, \tilde{H}, \widetilde{E}$, each of which is a function of x, ξ, t. The skewedness parameter $\xi \equiv r^+/2P^+$ can be expressed in terms of the Bjorken variable $\xi = x_{\mathrm{Bj}}/(2-x_{\mathrm{Bj}})$, but does not coincide with it. Depending on the value of x, each OFPD has 3 distinct regions. When $\xi < x < 1$, they are analogous to usual quark distributions; when $-1 < x < -\xi$ they are similar to antiquark distributions. In the region $-\xi < x < \xi$, the "returning" quark has a negative momentum, and should be treated as an outgoing antiquark with momentum $(\xi-x)P$. The total $q\bar{q}$-pair momentum $r=2\xi P$ is shared by the quarks in fractions $r(1+x/\xi)/2$ and $r(1-x/\xi)/2$. Hence, OFPD in this region $-\xi < x < \xi$ is similar to a distribution amplitude $\Phi(\alpha)$ with $\alpha = x/\xi$. In the local limit, OFPDs reduce to form factors

$$\sum_a e_a \int_{-1}^{1} H^a(x,\xi;t)\,\mathrm{d}x = F_1(t)\,, \tag{8}$$

$$\sum_a e_a \int_{-1}^{1} E^a(x,\xi;t)\,\mathrm{d}x = F_2(t)\,. \tag{9}$$

The E function, like F_2, comes with the r_μ factor, hence, it is invisible in DIS described by exactly forward $r=0$ Compton amplitude. However, the $t=0, \xi=0$ limit of E exists: $E^{a,\bar{a}}(x,\xi=0;t=0) \equiv \kappa^{a,\bar{a}}(x)$. In particular, its integral gives the proton anomalous magnetic moment κ_p, and its first moment enters into Ji's sum rule for the total quark contribution J_q into the proton spin

$$\sum_a e_a \int_{-0}^{1} (\kappa^a(x) - \kappa^{\bar{a}}(x))\,\mathrm{d}x = \kappa_p\,, \tag{10}$$

$$J_q = \frac{1}{2}\sum_a \int_{-0}^{1} x\,[f^a(x)+f^{\bar{a}}(x)+\kappa^a(x)+\kappa^{\bar{a}}(x)]\,\mathrm{d}x\,. \tag{11}$$

Note that only valence quarks contribute to κ_p, while J_q involves also sea quarks. Furthermore, the values of $\kappa_{p,n}$ (unlike $e_{p,n} \equiv F_1^{p,n}(0)$) strongly depend on dynamics, *e.g.*, $\kappa_N \sim 1/m_q$ in constituent quark models.

8 Double distributions

To model GPDs, two approaches are used: a direct calculation in specific dynamical models: bag model, chiral soliton model, light-cone formalism, etc., and a phenomenological construction based on the relation of SPDs to usual parton densities $f_a(x), \Delta f_a(x)$ and form factors $F_1(t), F_2(t), G_A(t), G_P(t)$. The key question in the second approach is the interplay between x, ξ and t dependencies of GPDs. There are not so many cases in which the pattern of the interplay is evident. One example is the function $\widetilde{E}(x,\xi;t)$ that is related to $G_P(t)$ form factor and is dominated for small t by the pion pole term $1/(t-m_\pi^2)$. It is also proportional to the pion distribution amplitude $\varphi(\alpha) \approx \frac{3}{4} f_\pi (1-\alpha^2)$ taken at $\alpha = x/\xi$. The construction of self-consistent models for other GPDs is performed using the formalism of double distributions [10].

The main idea behind the double distributions is a "superposition" of P^+ and r^+ momentum fluxes, *i.e.*, the representation of the parton momentum $k^+ = \beta P^+ + (1+\alpha)r^+/2$ as the sum of a component βP^+ due to the average hadron momentum P (flowing in the s-channel) and a component $(1+\alpha)r^+/2$ due to the t-channel momentum r. Thus, the double distribution $f(\beta,\alpha)$ (we consider here for simplicity the $t=0$ limit) looks like a usual parton density with respect to β and like a distribution amplitude with respect to α (fig. 7). The connection between the DD

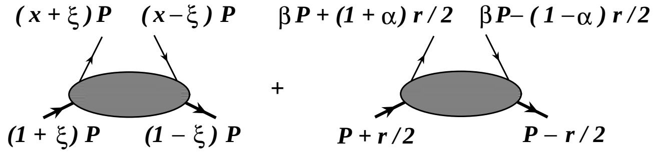

Fig. 7. Comparison of GPD and DD descriptions.

variables β, α and the OFPD variables x, ξ is obtained from $r^+ = 2\xi P^+$, which results in the basic relation $x = \beta + \xi\alpha$.

The forward limit $\xi = 0, t = 0$ corresponds to $x = \beta$, and gives the relation between DDs and the usual parton densities:

$$\int_{-1+|\beta|}^{1-|\beta|} f_a(\beta, \alpha; t = 0)\, \mathrm{d}\alpha = f_a(\beta)\ . \tag{12}$$

The DDs live on the rhombus $|\alpha| + |\beta| \le 1$ and they are symmetric functions of the "DA" variable α: $f_a(\beta, \alpha; t) = f_a(\beta, -\alpha; t)$ ("Munich" symmetry [11]). These restrictions suggest a factorized representation for a DD in the form of a product of a usual parton density in the β-direction and a distribution amplitude in the α-direction. In particular, a toy model for a double distribution

$$f(\beta, \alpha) = 3[(1 - |\beta|)^2 - \alpha^2]\, \theta(|\alpha| + |\beta| \le 1)$$

corresponds to the toy "forward" distribution $f(\beta) = 4(1 - |\beta|)^3$, and the α-profile like that of the asymptotic pion distribution amplitude.

To get usual parton densities from DDs, one should integrate (scan) them over vertical lines $\beta = x = \text{const}$. To get OFPDs $H(x, \xi)$ with nonzero ξ from DDs $f(\beta, \alpha)$, one should integrate (scan) DDs along the parallel lines $\alpha = (x - \beta)/\xi$ with a ξ-dependent slope. One can call this process the DD-tomography. The basic feature of OFPDs $H(x, \xi)$ resulting from DDs is that for $\xi = 0$ they reduce to usual parton densities, and for $\xi = 1$ they have a shape like a meson distribution amplitude. A more complete truth is that such a DD modeling misses terms invisible in the forward limit: meson exchange contributions and the so-called D-term, which can be interpreted as σ-exchange. The inclusion of the D-term induces nontrivial behavior in the central $|x| < \xi$ region (for details, see [5]).

9 Some lessons

Hadronic structure is a complicated subject, it requires a study from many sides, in many different types of experiments. The description of specific aspects of hadronic structure is provided by several different functions: form factors, usual parton densities, distribution amplitudes. Generalized parton distributions provide a unified description: all these functions can be treated as particular or limiting cases of GPDs $H(x, \xi, t)$.

Usual parton densities $f(x)$ correspond to the case $\xi = 0$, $t = 0$. They describe a hadron in terms of probabilities $\sim |\Psi|^2$. But QCD is a quantum theory: GPDs with $\xi \neq 0$ describe correlations $\sim \Psi_1^* \Psi_2$. Taking only one point $t = 0$ corresponds to integration over impact parameters $b_\perp$ —information about the transverse structure is lost.

Form factors $F(t)$ contain information about the distribution of partons in the transverse plane, but $F(t)$'s involve integration over the momentum fraction x —information about longitudinal structure is lost

Nonforward parton densities. A simple "hybridization" of usual densities and form factors in terms of NPDs $\mathcal{F}(x, t)$ (GPDs with $\xi = 0$) shows that behavior of $F(t)$ is governed both by transverse and longitudinal distributions. GPDs provide an adequate description of the nonperturbative soft mechanism, they also allow to study transition from soft to hard mechanism.

Distribution amplitudes $\varphi(x)$ provide quantum level information about longitudinal structure of hadrons. In principle, they are accessible in exclusive processes at large momentum transfer, when hard-scattering mechanism dominates. GPDs have DA-type structure in the central region $|x| < \xi$.

Generalized parton distributions $H(x, \xi; t)$ provide a 3-dimensional picture of hadrons. GPDs also provide some novel possibilities, such as "magnetic distributions" related to the spin-flip GPD $E(x, \xi, t)$. In particular, the structure of the nonforward density $E(x, \xi = 0, t)$ determines the t-dependence of $F_2(t)$. Recent JLab data give $F_2(t)/F_1(t) \sim 1/\sqrt{-t}$ rather than $1/t$ expected in hard pQCD and many models —a puzzle waiting to be resolved. The forward reductions $\kappa^a(x)$ of $E(x, \xi, t)$ look as fundamental as $f^a(x)$ and $\Delta f^a(x)$: Ji's sum rule involves $\kappa^a(x)$ on equal footing with $f(x)$. Magnetic properties of hadrons are strongly sensitive to dynamics, thus providing a testing ground for models. Another novel possibility is the study of flavor-nondiagonal distributions, *e.g.*, proton-to-neutron GPDs accessible through processes like exclusive charged-pion electroproduction, proton-to-Λ GPDs (they appear in kaon electroproduction); proton-to-Delta —this one can be related to form factors of the proton-to-Delta transition (another puzzle for hard pQCD). The GPDs for $N \to N + \text{soft } \pi$ processes can be used for testing the soft-pion theorems and the physics of chiral-symmetry breaking.

An interesting problem is the separation and flavor decomposition of GPDs. The DVCS amplitude involves all 4 types: $H, E, \widetilde{H}, \widetilde{E}$ of GPDs, so we need to study other processes involving different combinations of GPDs. An important observation is that, in hard electroproduction of mesons, the spin nature of the produced meson dictates the type of GPDs involved, *e.g.*, for pion electroproduction, only $\widetilde{H}, \widetilde{E}$ appear, with $\widetilde{E}$ dominated by the pion pole at small t. This gives access to (generalization of) polarized parton densities without polarizing the target.

10 Summary and conclusions

The structure of hadrons is the fundamental physics to be accessed via GPDs. GPDs describe hadronic structure on the quark-gluon level and provide a 3-dimensional picture ("tomography") of hadronic structure. GPDs adequately reflect the quantum-field nature of QCD (correlations, interference). They also provide new insights into spin structure of hadrons (spin-flip distributions, orbital angular momentum). GPDs are sensitive to chiral-symmetry breaking effects, a fundamental property of QCD. Furthermore, GPDs unify existing ways of describing hadronic structure. The GPD formalism provides nontrivial relations between different exclusive reactions and also between exclusive and inclusive processes.

I thank the organizers for invitation, support and hospitality in Elba. This work is supported by the US Department of Energy contract DE-AC05-84ER40150 under which the Southeastern Universities Research Association (SURA) operates the Thomas Jefferson Accelerator Facility.

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